ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-V

Paper Code: C11T

[Partial Differential Equations and Application]

Answer all the questions

1. From a PDE when $\varphi(u, v) = 0$, where $u = x + y + z,\ v = x^2 + y^2 + z^2$.
2. Define quasi-linear and semi-linear partial differential equation.
3. Define 'Dirichlet boundary condition' and 'Neumann boundary condition'.
4. Give the geometrical interpretation of Cauchy IVP $u_t + cu_x = 0, x \in R, t > 0$ where $u(x, 0) = f(x),\ x \in R$.
5. Write the different types of first order PDE with standard form.
6. Show that the solution of the PDE: $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ is of the form $f({}^{y}/_{x})$.
7. A particle describes a curve $s = c\ tan\psi$ with uniform speed v. Find the acceleration indicating its direction.

8. What is ballistics? Write different types of ballistics.
9. Prove that a planet has only radial acceleration towards the Sun.
10. Let $u(x,t)$ be the solution of the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ with $u(x,0) = \cos(5\pi x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. Then prove that $u(1,1) = 1$.
11. Find the characteristic curve of PDE: $2y\frac{\partial u}{\partial x} + (2x + y^2)\frac{\partial u}{\partial y} = 0$ which is passing through the point (0, 0).
12. Prove that at an apse on a central orbit, the velocity is proportional to the reciprocal of the radius vector.
13. Find PDE corresponding to the equation $z = xy + f(x^2 + y^2)$, f being an arbitrary function.
14. What is the nature of the second order PDE $\frac{\partial^2 z}{\partial y^2} - y\frac{\partial^2 z}{\partial x^2} + xz = 0$?
15. If a particle moves on a curve $\sqrt{r}\cos\frac{\theta}{2} = \sqrt{a}$ with cross-radial velocity constant then show that the velocity of the particle is constant.
16. Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the following conditions : $u(0,t) = u(2,t) = 0$, $u(x,0) = \sin\frac{\pi x}{2}$, $u_t(x,0) = 0$.
17. A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\tan^{-1}\sqrt{\frac{4ag}{V}}$.
18. Find the complete integral of the PDE $z^2 = \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}xy$, by Charpit's method.
19. A particle describes an ellipse under a force $\frac{\mu}{(distance)^2}$ towards the focus; if it was projected with velocity V from a point distance r from the centre of the force, show that its periodic time is

20. Solve the following: $(D^2+5DD'+D'^2)z=0$ where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$.
21. Find the solution of $z^2=pqxy$.
22. Solve: $(x^2D^2-2xyDD'+y^2D'^2-xD+3yD')u=8\frac{y}{x}$. Symbols have their usual meaning.
23. Establish the Laplace equation in polar coordinates.
24. Find the solution of $(D^2-DD'-2D'^2)z=(y-1)e^x$, Where $D=\frac{\partial}{\partial x}$ $and\ D'=\frac{\partial}{\partial y}$.
25. Solve the following PDE: $(x^2D^2-4xyDD'+4y^2D'^2+6yD')z=x^3y^4$ Where $D=\frac{\partial}{\partial x}$ $and\ D'=\frac{\partial}{\partial y}$.
26. Find the integral surface of the linear PDE $(x-y)p+(y-z-x)q=z$ which passes through the circle $x^2+y^2=1, z=1$.
27. Reduce the following equation to a canonical form and hence solve it $yu_{xx}+(x+y)u_{xy}+xu_{yy}=0$.
28. Find the general integral of the PDE $p^2y(1+x^2)=qx^2$.
29. Establish the d' Alembert's formula for solve the Cauchy problem for homogeneous wave equation.
30. Find the PI of the PDE $(D^2+DD'+D'-1)z=\sin(x+2y)$.
31. What are the main difference between an ODE and PDE ?
32. Eliminate the arbitrary function f and F from y = f(x-at) + F(x+at) .

33. Solve the problem $u_{tt} - u_{xx} = 0 \quad 0 < x < \infty,\ 0 < t$, $u(0,t) = \frac{t}{1+t}, \quad 0 \le t\,,\ u(x,0) = u_t(x,0) = 0,\ 0 \le t < \infty$.

34. Find the solution of the following problem $u_x + x^2 u_y = 0$ with $u(x,0) = e^x$.

35. Let $u(x,y)$ solve the Cauchy problem $\frac{\partial u}{\partial y} - x\frac{\partial u}{\partial x} + u - 1 = 0$ where $-\infty < x < \infty,\ y \ge 0$ and $u(x,0) = \sin x$. Then find the value of $u(0,1)$.

36. Consider the initial value problem $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(0,y) = 4e^{-2y}$. Then find the value of $u(1,1)$.

37. Find the complete integral of the PDE $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = xe^{x+y}$.

38. Let P(x, y) be a particular integral of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 2y - x^2$. Then find the value of P(2, 3).

39. Let u be the unique solution of $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, x \in R, t > 0,\ u(x,0) = f(x), \quad \frac{\partial u}{\partial t}(x,0) = 0,\ x \in R$. Where $f(x) = x(1-x), \forall x \in [0,1]$ and $f(x+1) = f(x) \forall x \in R$. Then find the value of $u(\frac{1}{2},\frac{5}{4})$.

40. Let u(x, t) satisfy the initial boundary value problem $u_t = u_{xx}\,; x \in (0,1), t > 0,$ $u(x,0) = \sin(\pi x)\ ; x \in [0,1]$, $u(0,t) = u(1,t0 = 0, t > 0$. Then find the value of $u\left(x, \frac{1}{\pi^2}\right),\ x \in (0,1)$.

41. Let u(x, t) be the solution of $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = xt, \quad -\infty < x < \infty, t > 0$,

$u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0, \quad -\infty < x < \infty$. Then find the value of u(2, 3).

42. Let u(x, t) satisfy the initial boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty, t > 0 \qquad u(x,0) = \cos(\frac{\pi x}{2}) \ , 0 \leq x < \infty$ $\frac{\partial u}{\partial t}(x,0) = 0, 0 \leq x < \infty \ , \frac{\partial u}{\partial x}(0,t) = 0, \ t \geq 0$. Then find the value of $u(2,2)$ and $u(\frac{1}{2}, \frac{1}{2})$.

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